4.3: Matrix Inversion

Definition 1. The inverse of an $n \times n$ matrix A is the $n \times n$ matrix A^{-1} such that

$$AA^{-1} = A^{-1}A = I.$$

If the inverse of A exists, it is said to be invertible. Otherwise, it is said to be singular.

Example 1. Is $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ invertible?

Example 2. Compute the inverse of the following matrices.

(a) $P = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

(b) $Q = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 3 & 0 & 0 \end{bmatrix}$

Theorem 1. The inverse of a 2×2 matrix is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix},$$

provided $ad - bc \neq 0$. The quantity $ad - bc$ is called the <u>determinant</u> of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If the determinant is 0, then the matrix is singular.

Example 3. Determine if the matrix $S = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & 4 \\ 3 & 1 & -2 \end{bmatrix}$ is invertible by looking at the reduced row echelon form.

Example 4. Solve the system of equations

$$2x + z = 1, \quad 2x + y - z = 1, \quad 3x + y - z = 1$$

using matrix inversion.